In the Club: Strategic Admission, Membership, and the Endogenous Splitting of Clubs *

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Abstract

Much of the work on club goods has been of a static nature, which potentially distorts many strategic options of the members of clubs. I show that when forward-looking individuals have horizontally heterogeneous preferences over the provision of the club good, those whose preferences are furthest from that of the club may split and form a new club. These individuals may not split immediately, instead delaying and incubating this deviant club in order to take advantage of scale economies. While clubs cannot fully prevent this behavior, they may choose to limit (encourage) it by either charging an entry fee (bonus), or by assigning property rights such that those who split leave with nothing (everything).

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1 Introduction

Communities and clubs play a central role in many branches of economics, including political economy, industrial organization, the economics of innovation, and religion, to name a few. For example, members of a political party work together to raise capital. Other examples include religious clubs, open source software, as well as some firms.[1] This paper develops a dynamic model with equilibria that are consistent with much of the observed behavior in membership decisions and production of club goods that is distorted in static models, with a focus on the endogenous formation and splitting of these clubs. The primary mechanism behind the formation and fracturing of clubs is the tradeoff between enjoying the benefits from local public goods and individual preference heterogeneity, coupled with depreciation and discounting.

Following Buchanan (1965), I define a club as a group of individuals who contribute to produce a good that is both non-rival and non-excludable to members, but excludable to non-members. Consider the following illustration of the tenure of employees at consulting firms. Many individuals who join consulting firms do not remain for the entirety of their career. According to wetfeet.com, "a fair number of consultants will leave the business after two or three years to pursue entrepreneurial or industry positions." Furthermore, each consulting firm differ in their corporate culture, focus, and approach.[2]

In 1973, a group of ten individuals founded Bain & Company (Bain), one of the largest consulting companies in the world.[3]. These individuals did not simply appear out of the woodwork, but were in fact employees of a different consulting firm, Boston

[1] A firm where employees are compensated by both commission and profit sharing could be interpreted as a club, e.g., a consulting firm.

[2] https://www.wetfeet.com/articles/career-overview-management-consulting, accessed April 21, 2014.

[3] http://www.bain.com/about/what-we-do/history-of-innovation.aspx, accessed April 21, 2014

Consulting Group (BCG). Bain was not simply a carbon copy of BCG, but implemented different practices and a different corporate culture. When founded, Bain implemented a one-client-per-industry rule to prevent conflicts of interest (Naficy, 1997), partners did not carry business cards, clients were referred to by codes to enforce confidentiality, and clients were acquired not through marketing, but through boardroom referrals (Sweeney, 2001).

Thus we have the following scenario. There are several consulting firms, one of which being BCG, with each firm having its own corporate culture. In 1973, ten BCG employees decided to form a new consulting firm, Bain, with its own corporate culture, distinct from that of BCG. The underlying question is why did the employees choose to work for BCG at all, as opposed to forming Bain instead of working at BCG? Furthermore, why did BCG allow these individuals to form Bain and locate it in Massachusetts, where BCG is also located?

I use a two period framework to address the issue, focusing on four key research questions:

1) Why do clubs split?

2) Conditional on splitting, how is the timing of the split determined?

3) From the viewpoint of the club, knowing that some members may leave, why let them in at all?

4) Can the club control this splitting behavior?

This paper posits that the driving force behind the splitting of clubs is horizontal heterogeneity of preferences among individual members. Given the non-rival, non-congestible nature of the club good, increasing membership has a positive individual effect. There are more members to share in the cost of provision and there are (potentially) economies of scale and increasing returns in production. The negative of increasing membership is an increased incentive for free-riding, and thus the distance between the Nash equilibrium level of contribution and the socially optimal level is nondecreasing in the number of members; however, individual utility is still (generally) increasing in the number of members.[4] Why then, might individuals split? If individuals have heterogeneous preferences, where different members have different ideals about what the characteristics of the club good should be, how the good should be provided, etc., then a subset of members face the incentive to leave and form a new club, where the new club's agenda is a closer match to the individuals' perceived "ideal" agenda. Thus the members who split give up some the benefits associated with club size in exchange for a club good that is intrinsically more valuable at every level of provision.

Given that some members have the incentive to split under heterogeneity, how do these members time their splitting decision? These individuals will strategically wait and pick a time which maximizes the (expected) discounted future stream of payoffs. This optimal time may or may not be at the outset. The timing of the split is determined by what I call the *contribution-scale tension*, which represents the size versus intensity of contributions tradeoff, and the *agenda-scope tension*, which represents the preference-based marginal utility tradeoff.[5] If the magnitude of the contribution-scale tension outweighs the magnitude of the agenda-scope tension, then the phenomenon of *strategic membership* occurs, where individuals join clubs with the

[4]Consider the simple example where $u_i(x_i, x_{-i}) = \sum_{j=1}^{N} x_j - \frac{1}{2}x_i^2$. The Nash equilibrium is $x_i^* = 1$, $\forall i$. Thus, each individual's indirect utility function is $N - \frac{1}{2}$, which is strictly increasing in N. The socially optimal level of contribution is $\hat{x}_i = N$, $\forall i$, implying an indirect utility of $\frac{1}{2}N^2$, which is also increasing in N, as is the difference in the socially optimal and Nash equilibrium payoff. For more details, see Cornes and Sandler (1996).

[5]I provide more formal definitions in section 2.

intent to split in the future. If there is much to gain from being a member of a larger club that doesn't match one's preference, then individuals will sacrifice utility today for larger gains in the future. This outcome is distorted in static models, where the decision splitting decision is equivalent to joining: join or do not join.[6] Similar behavior could be observed (under certain assumptions) in a static model in which there are multiple club (or public) goods, in which one of the goods acts as an input to the other; however, this still distorts some of the more interesting timing dynamics. What would be observed, rather than splitting, are contributions to both goods by an individual.

From the club's point of view, there may be an incentive to reject the membership (if possible), of those individuals who plan on leaving in the future. Clubs can be decomposed into two types - centralized and decentralized, where centralized clubs act in a utilitarian fashion and decentralized clubs act as individuals.[7] A decentralized club benefits from increasing the number of members since each person's marginal contribution will decrease while the overall level of contributions increase.[8] Thus the decentralized club would always face the individual incentive to let members in, even if those members plan on leaving. In centralized clubs, the total level of contributions is a function of the average marginal cost and average marginal benefit. When adding individuals whose marginal benefit is well below average or whose marginal cost is well above average, the total level of contributions may decrease and this can lead to a decrease in utility for the individual members. Thus the club faces an incentive to practice *strategic admission*, where, for example, the club charges an entree fee to guarantee that only members with preferences near that of the club join. For the

[6]The same is true for myopic models of club goods contribution.

[7]This idea was originally introduced in Olson (1965).

[8]Note that adding members may be inefficient, though I do not explicitly address this issue. Aimone et al. (2013) illustrate a selection mechanism that addresses this type of inefficiency.

remainder of the paper, I consider decentralized clubs and thus consider subgame-perfect equilibria. It should be noted that the main results are dependent on the indirect utility function, and not directly on the method of contribution, so the main results can also be directly applied to centralized clubs.

If the club is unable to practice strategic admission, it still may have an option to prevent strategic membership. Let us reexamine the Bain example. In order for strategic membership to occur, there must be a reason for the founders of Bain to incubate the deviant club (Bain) within the parent club (BCG). Those individuals contributed human capital to BCG, and in return, received back both human capital and a reputation within the industry. Thus the members initiating the split are free to take any and all of the good with them (their human capital and reputation). I broadly define the contributions that can be taken as ritual. Suppose that the club (or the government) can control the movement of the good, e.g. property rights or by including ancillary requirements for any subset of the ritual. Then the club can control how much can be taken when the club splits - and can choose a level such that the club (or government) can thus control whether or not strategic membership occurs.

Clubs may not be able to prevent the initial split from occurring at the outset. If there exists a profitable split at the outset, then in order to prevent the split, the club would need to compensate those with the desire to split with, at minimum, the gains from splitting. This can be both costly and infeasible without an external enforcement mechanism.

This paper provides formal results for the ideas expressed above. The remainder of the paper is structured as follows. I first provide a brief review of the related literature. Section 2 provides several applications. Section 3 details the explicit model and results, as well as both welfare and empirical implications. Section 4 provides some extensions and future directions to consider and section 5 concludes.

1.1 Literature Review

The literature on the theory of club goods can be traced back to two key works, Buchanan (1965) and Olson (1965). The first models in the theory of club goods were nonstrategic, in that the analysis was typically done considering welfare rather than what are now considered standard game theoretic tools and concepts (e.g. Nash equilibrium). Buchanan formally introduced the idea of a club good - a good located in an intermediate position on the spectrum from purely private goods to purely public goods while Olson's book provided a treatment of many of the aspects of clubs themselves, rather than the goods they produce. Since those early works, much has been done to advance various branches of the literature. A comprehensive review of this is outside of the scope of this paper, so I focus instead on a small subset of the relevant literature. A more thorough review of the literature is available in Cornes and Sandler (1996).

This paper contributes to the literature on the private provision of public goods, club goods, as well as the small but growing literature on dynamic public and club goods and the splitting of clubs. Further, this paper contributes to the literatures regarding the relevant applications, such as the economics of religion, industrial organization, and political economy. The private provision of public goods literature can be traced back to Bergstrom et al. (1986) and Bernheim (1986). Since then, much has been done on voluntary contributions. For literature reviews on public and club goods, see Cornes and Sandler (1996), Sandler and Tschirhart (1997), and Scotchmer (2002).

A complete formal model of endogenous club formation is developed in Haimanko et al. (2004), where horizontally heterogeneous individuals partition themselves into communities.[9] Polborn (2008) considers a simplified formation process where there already exists a set of clubs located around the unit circle. New individuals arrive and make membership decisions. Similar applied models have been developed analyzing nation building, e.g. Alesina and Spolaore (1997). Through experiments, Ahn et al. (2008) analyze how rules, such as entry and exit fees, impact endogenous club formation.[10] They find that these rules do have significant impacts on economic outcomes, a result consistent with this paper. However, none of these papers have considered time dynamics, where the economic incentives may vary from period to period. Furthermore, excluding the literature on nation building, the aforementioned work does not spend much time considering the splitting of preexisting groups. One branch of literature that has devoted significant energy to this is the economics of religion.

The dynamic literature on club goods can be traced back to conjectures made in Becker (1974). Five years later, McMillan (1979) analyzed the free-rider problem in the context of a repeated game. It wasn't until 1991 that Fershtman and Nitzan (1991) verified Becker's claim on dynamic inefficiency and the free-rider problem in linear strategies. This was extended to nonlinear strategies in Wirl (1996). Over the past 15 years, there have been only a few papers related to analytic models with dynamic contributions, such as Glomm and Lagunoff (1999), Marx and Matthews (2000), and Arnold and Wooders (2005), where the focus of these papers is on the voluntary v. involuntary contributions (i.e. decentralized v. centralized clubs) and long run outcomes. In Polanski (2007), a dynamic model of software contribution is considered, where contributions are a public good and can thus be applied to club

[9]For a a model with vertical heterogeneity, see Jaramillo et al. (2003).

[10]See citations within Ahn et al. (2008) for a literature review focusing on experimental evidence for endogenous groups.

goods; however, the paper explicitly assumes splitting does not occur.

One applied area of club economics has been exceptionally keen in incorporating splits and dynamics, though typically not together, is the economics of religion (e.g. the Protestant Reformation). Iannaccone (1992, 1994) spearheaded the literature, which included modeling contributions and reducing free-riding in collectives. Several papers, including Makowsky (2011, 2012), employ agent-based modeling to capture dynamics in contributions. A recent paper beginning to explore the modeling of schisms is Maloney et al. (2010). While Maloney et al. serves as a good first step in analyzing schism, it lacks the formal structure necessary for generalizability.

2 Three Applications

In this section, I provide three examples (not including the aforementioned consulting example) which illustrate the both the scope of what can be explained by the model as well as the breadth of applications the model can be applied to. I have abstracted away from much of the detail in order to illustrate the key similarities.

2.1 Political Parties

Let us consider a political party from the viewpoint of politicians currently in office, as opposed to the standard viewpoint of candidates. Once in office, politicians have more freedom than they would as a candidate (stemming from rational ignorance on the part of the voter). Every political party has a well defined agenda - a set of ideals and goals which its members attempt to achieve through some process, typically via legislation. Each politician also has their own individual set of ideals. In order to take advantage of the system, politicians join the party whose agenda best matches with their own, but is that enough? Even though the politician joins the party more in tune with his/her ideals, what if there is still a large gap between the two?

Consider the Republican party in the United States (GOP). The GOP's agenda can be described, in the broadest sense, as conservative. However, the "conservative" label would be inadequate in describing the agendas of elected officials in the GOP. There are members across the political spectrum ranging from moderate, conservative-constitutionalist, to libertarian. This heterogeneity has lead to much strife within the party.[11]

What appears to have occurred is a "fracturing" of the GOP. While the party still formally remains a single entity, there are two distinct groups within the parent party: the establishment and the Tea Party. The Tea Party challenges the establishment in many of the primaries and has a distinct agenda, further away from the middle of the political spectrum than the establishment. Political movements can also be captured by this type of behavior, such as the Socialist Party schism of 1919 (Miller, 1995).

2.2 Open Source Software

Open source software (OSS) is a prime example of collective innovation at work. Individuals collaborate to develop computer software, whose source code is open in nature. That is, the code is made freely available (to a varying degree) to anyone. If we isolate the world to the community of individuals who contribute to the development of OSS, an interesting phenomenon is observed.

First, there exists many substitutes for software performing a given function, e.g., Linux operating systems. There are Ubuntu and Lubuntu, which are derivatives of Debian. Ubuntu is one of the more popular Linux Distributions, ranking number two in the list of top ten distributions made by distrowatch.com.[12] Mark Shuttleworth

[11]E.g. http://www.dpcc.senate.gov/?p=blogid=255.
[12]http://distrowatch.com/dwres.php?resource=major, accessed April 21, 2014.

and a small team of developers took the Debian source code in order to develop an operating system and a new community associated with it, and the first Ubuntu distribution was released in 2004.[13] A few years later, a group of developers within Ubuntu wanted to develop a variant of Ubuntu which was lighter and more suitable for slower or older computers. The Lubuntu project took Ubuntu's original source code and did just that.[14]

2.3 Religious Schisms

The 16th century Protestant Reformation is a topic studied by many in the economics of religion literature. A movement, spearheaded by reformers including Martin Luther, John Calvin, and Henry VIII, challenged the Catholic church. They argued for changes including ritual, doctrinal interpretation, and philanthropic efforts. While this lead to much conflict, the ultimate result was a schism in which there was no longer a unified church within Western Christianity.

3 The Model

Consider an environment where there are two time periods, indexed by $t = 1, 2$. Payoffs in the second period are discounted by common factor $\delta \in [0, 1]$, and there exists a community I, consisting of a continuum of individuals with unit mass.[15] Each individual $i \in I$ of the community has a "preferred agenda" (henceforth referred to as an agenda for short) $A_i \in \mathbb{R}$. Suppose that there are two types of individuals in the community, labeled type-a and type-b, each having respective agenda α and β, with $\alpha \neq \beta$. Furthermore, suppose that a proportion $\lambda \in (0, 1)$ of the community is

[13] http://www.ubuntu.com/about/about-ubuntu, accessed April 21, 2014.

[14] https://wiki.ubuntu.com/Lubuntu, accessed April 21, 2014.

[15] Note all results apply for large groups of finite size N as well. The only changes are in notation.

of type-a while the remaining $1-\lambda$ is of type-b.

Along with the community, there is a set of clubs $\mathcal{K}$. Each club $k \in \mathcal{K}$ has its own agenda A_k. The cardinality of $\mathcal{K}$ is endogenously determined by the community in a manner to be specified; however, for there to exist multiple clubs, for any two clubs k and k', it must be that $A_k \neq A_{k'}$. A club can be technically thought of as a partition of the community, implying that individuals can only be a part of a single club.[16] I denote an individual i to be a member of club k at time t ($i_t \in k$) if she contributes to the good produced by k during period t.

Each contributing member of a club receives two types of benefits - a global benefit and a local benefit. That is, each member receives utility from both the overall level of contributions and from her own personal contributions. For example, in open source software, contributors receive a benefit from the software itself, which is a function of all member contributions, and a benefit from their own contributions. A local (private) benefit could be a signaling payoff, where contributing to the code is a positive signal of skill to potential employers.[17] With respect to consulting, the local benefit can be interpreted as commission, human capital development from experience, and reputation, while the global benefit can be interpreted as gains in human capital and reputation through positive externalities (interacting with others), as well as revenues from profit sharing. Let $C_t^k(i)$ denote member i's marginal contribution of capital to club k at time t, $C_t^k = \int_{j_t \in k} C_t^k(j)dj$ denote the total marginal contribution to club k at time t. Suppose that capital decays at rate $\gamma \in [0,1]$.[18] Thus club $k's$ level of capital (installed base) at time $t=1$ is $IB_1^k = C_1^k$ and at $t=2$, the installed base is $IB_2^k = (1-\gamma)C_1^k + C_2^k$. Define $G(IB_t^k; A_i - A_k)$ as the global benefit for individual i if she is a contributing member of club k and $L(C_t^k(i); A_i - A_k)$ as her local benefit. Contribution comes at a cost, denoted by $F(C_t^k(i))$. Thus the general utility function

[16] This is consistent with Haimanko et al. (2004).

[17] For more detail on this breakdown in open source software, see Lerner and Tirole (2002).

[18] This is akin to the differential game setup by Fershtman and Nitzan (1991).

for member i of club k is defined as

$$U_{it,k} \equiv U(C_t^k(i); IB_t^k, A_i, A_k) = G(IB_t^k; A_i - A_k) + L(C_t^k(i); A_i - A_k) - F(C_t^k(i)). \quad (1)$$

For technical purposes, I assume that G, L, and F jointly satisfy three properties - that $U_{it,k}$ is strictly quasi-concave, that G is weakly concave in contributions, and that G, L, and F are continuously differentiable with respect to capital.[19] Furthermore, suppose that both G and $\frac{\partial G}{\partial C_t^k(i)}$ are decreasing in $|A_i - A_k|$. I assume that the same holds for L and $\frac{\partial L}{\partial C_t^k(i)}$. This implies that both utility and marginal utility are increasing in how well matched an individual's agenda is to the club's agenda.

Splitting from a club to form a new one is costly. The cost of breaking away from the club can be divided into two types - a social cost and a contribution cost. The social cost can be thought of as the price to pay if splitting is considered "taboo," e.g. a stigma. Alternatively, this can be thought of as the price of either forming a new agenda, or changing an existing agenda. I assume that this cost is fixed for each split, but decreasing in the number of splits.[20] Let S represent the number of splits that have already occurred. Then the social cost of splitting can be written as $f(S)$, where $f(0)$ is the social cost of the first split, and $f(0) \geq f(1) \geq \ldots$. I assume that there is no cost of switching clubs outside of the change in utility given the new installed base.

When splitting, there may be some loss incurred. Let $\rho(|A_k - A_{k'}|)$ represent this loss function, where $A_{k'}$ represents the agenda of the new club k'. Assume that $\rho(\cdot) : \mathbb{R} \to [0, 1]$ and is nonincreasing in $|A_k - A_{k'}|$. Therefore, when splitting at time $t+1$, the new club is able to bring $(1-\rho(|A_k - A_{k'}|))(1-\gamma)IB_1^k$ from the original club.

[19] This ensures a unique solution under various mechanisms, such as individual utility or welfare maximization.

[20] This assumption is based on Thomas Schelling's tipping point model (Schelling, 1978) and Kuran (1987, 1995).

It is also possible to consider ρ as a strategic variable. For example, suppose that the club good represents an innovative effort. ρ could be interpreted as the strictness of IP. If clubs are able choose ρ, then they could exert control over the splitting decisions. I expand upon this further in a subsequent subsection (3.5), but for now, assume ρ is defined by the above function.

The final pieces to consider are the informational assumptions. There are three possible sources of uncertainty: the individual marginal contributions $C_t^k(i)$, the potential clubs A_k, and the individual agendas A_i. Given the modern state of communication technology, it is fairly innocuous to assume that all agendas are known by all individuals, both at the individual level and club level. Since the agendas are all known, the payoff functions for all agents are known which in turn implies that the contributions are known. Thus for the purposes of this paper, I assume there is no uncertainty. Note that this assumption could be relaxed and expectations over the distributions of agendas could be formed. Furthermore, preferences could change overtime - another condition which I do not consider. These complications are left for future work.

3.1 The Dynamics

At the outset, suppose that there is an initial club, with agenda $A_0 = \lambda\alpha + (1-\lambda)\beta$. Contributions occur repeatedly over time with the environment proceeding as follows. At time $t = 0$, a single club is formed with agenda A_0, and all individuals are members of the club. At each time $t = 1, 2,$ each member i can choose whether to continue contributing to the existing club (status quo), leave the status quo and begin a new club, or if multiple clubs currently exist, switch clubs. Members are restricted to only contributing to a single club.[21]

3.2 Baseline Results

Suppose, for comparison, that splitting is not allowed. In this case, the model represents an environment in which individuals contribute to a single club for two periods. Type-i's maximization problem can be represented by the system:

$$\begin{aligned}
&\max_{C_1(i)}\{G(C_1; A_i, \lambda d + (1-\lambda)r) + L(C_1(i); A_i, \lambda d + (1-\lambda)r) - F(C_1(i)) \quad (2)\\
&\qquad + \delta[G((1-\gamma)C_1 + C_2; A_i, \lambda d + (1-\lambda)r) + L(C_2(i); A_i, \lambda d + (1-\lambda)r)\\
&\qquad - F(C_2(i))\},\\
&\max_{C_2(i)}\{G((1-\gamma)C_1 + C_2; A_i, \lambda d + (1-\lambda)r) + L(C_2(i); A_i, \lambda d + (1-\lambda)r) \quad (3)\\
&\qquad - F(C_2(i))\}.
\end{aligned}$$

At $t = 1$, each individual chooses $C_1(i)$ to maximize the total discounted utility. Then at $t = 2$, each individual chooses $C_2(i)$ to maximize the second period utility, given the choice of $C_1(i)$. Thus a simple exercise in backwards induction can yield the characteristics of the unique type-symmetric (each type plays the same strategy, which may differ across types) subgame-perfect equilibrium.

Proposition 1. *Define the equilibrium contribution level as $C_t^*(i)$ for $t = 1, 2$, $i \in \{a, b\}$. $C_t^*(a) \geq C_t^*(b)$ if and only if $\lambda \geq \frac{1}{2}$. Furthermore, both types contribute less than they would if there was a homogeneous population.*

[21] This is primarily for analytical tractability; however, one could think of the groups as close enough substitutes that the marginal benefit of joining a second club is so small that it is unprofitable to do so, or equivalently, budget constraints prevent it.

The proof, along with all subsequent proofs, can be found in the appendix. At first glance, this result seems trivial - the majority group (weakly) contributes more to the club than the minority group. Closer inspection leads to a more interesting result. Each individual in the majority group (weakly) contributes more than each individual in the minority group. Given that the agenda of the club is a weighted average of the agendas of the two types, the agenda will be a closer match to the majority type. Further, since the club's agenda splits the difference between the agendas of the various types, each type discounts the marginal benefits by their respective distance from the implemented agenda. This is consistent with Alesina et al. (1999), who show that the level of provision of some local public goods, such as education, roads, and sewers, is inversely related to the degree of ethnic fragmentation. In groups where there is significant heterogeneity, differences in preferences (between ethnic groups), decrease provision since individuals whose preferences are further away discount the benefits of provision while still bearing the costs. Now that a baseline for comparison has been established, I proceed to introduce splitting into the model.

3.3 The Two Tensions of Splitting

There are two tensions that guide the direction of the model, determining decisions at both the extensive and intensive margin. Jointly, these tensions will determine both whether or not splits occur and how they occur. The first component to consider is club size and club relevance. At the outset, every individual is contributing to the same club. Note that the club good has the characteristics of a local public good for the members. That is, there is a large benefit of having many people contribute to a single club. However, this comes at a cost. Many of the individuals will be contributing to a club whose agenda is relatively far away from their preferred agenda, which means those individuals are not willing to contribute as much, which could further diminish the contributions of other agents.[22] This implies that there is a tradeoff of the utility gains from scale economies and the loss from having a lower marginal benefit of contributing. Thus we have our first tension.

Tension 1. (The Contribution-Scale Tension) *When splitting, there is a tradeoff of the decrease in club size and the gains of more effective contributions. That is, the number of contributors decreases, but each contributor may be willing to contribute more.*

Furthermore, forming a new club leads to some capital losses proportional to the distance between the old and new agendas. The more distance that is placed between the new agenda and the old agenda, the more likely it is that some of the capital being brought over in the split depreciates in value. For example, if considering a split to be a religious schism, some rituals may become moot under the new agenda (doctrine).[23] Thus we have our second tension.

Tension 2. (The Agenda-Scope Tension) *Both the payoff and the cost of orchestrating a split are increasing in the distance between the old and new agenda. That is, the loss of contributions is increasing in the distance, but the marginal value of a contribution is increasing as well.*

[22]For example, if the mechanism to determine the overall contribution is a welfare maximization problem with individual contributions being symmetric, then having an individuals with lower marginal benefits decreases the overall level, which in turn decreases the individual contributions.

[23]E.g. Maloney et al. (2010).

The relative magnitude of the above two tensions will determine the outcome of the club. The first tension can be thought of as determining the extensive margin - whether or not a split will occur. The second tension represents the intensive margin - where the agenda of the new club will form. It is worth noting that these two effects cannot be analytically decoupled from each other whenever there is a contribution loss from splitting, viz. unless $\rho(\cdot) = 0$ at all values.

These tensions are described in the political economy literature, albeit informally, when analyzing nation building. Alesina and Spolaore (1997) discuss the tradeoff between nation size and the cost of heterogeneity in large populations. Before explicitly considering a split, let us begin by analyzing a key implication of a split occurring.

Proposition 2. *If a coalition of individuals are orchestrating a split, they will contribute more than if they planned to remain in the original club. Furthermore, those who are not splitting decrease their contributions; however, the cumulative decrease by the non-splitting group is less than the cumulative increase by the splitting coalition.*

Mathematically, the marginal benefit increases under a split while the marginal cost function remains unchanged whether or not a split is planned. The intuition is straight forward when considering a split at $t = 1$. The coalition orchestrating the split form a club whose agenda matches their own. Thus the marginal benefits from contributing are larger at all contribution levels. The more interesting case is when the split occurs at $t = 2$, since contributions by those who split at both $t = 1$ and $t = 2$ are larger. This follows from forward looking behavior. At time $t = 1$, individuals consider how their period one contributions impact the period 2 installed base. Since the marginal value of all contributions in period two increase, individuals are induced to increase period one contributions.

Those who don't split are able to free ride of off the increased contributions by those who do split. Since the splitting individuals have increased their contribution levels, those who do not split are able to decrease their contributions without losing any utility. Now that we have outlined some implications of splitting, the next step is to characterize the extensive margin, that is, when splitting will occur.

Proposition 3. *A split will occur under combinations of the following conditions:*

(i) $|\lambda - \frac{1}{2}| \geq \xi_{1,T} > 0$*, where* T *represents the time at which the split occurs* ($T \in \{1, 2\}$).

(ii) $||\alpha| - |\beta|| \geq \xi_{2,T}$.

(iii) $F''(\cdot)$ *is bounded above by some value* $\xi_{3,T} > 0$ *on the interval* $[0, \bar{C}]$*, where* $\bar{C}$ *is the largest equilibrium value of* $C_t(i)$.

(iv) $|\frac{\partial^2}{\partial C_t(i)^2} x(\cdot; \cdot)|$ *is bounded below by* $\xi_{4,T} > 0$ *on the interval* $[0, \bar{C}]$*, for* $x = G$ *or* L.

The values of $\xi_{xT}, x \in \{1, 2, 3, 4\}$ and $\bar{C}$ are defined in the appendix with the proof. Thus there are four non-mutually exclusive conditions under which splitting can occur. The first two can be attributed to the agenda-scope tension while the last two can be attributed to the contribution-scale tension. The first condition represents an imbalance of types. If one type is an extreme minority, then the initial club's agenda will be much more tailored towards the majority type (since the agenda is $\lambda\alpha + (1 - \lambda)\beta$). This implies the minority type has much to gain from forming a new club with a tailored agenda.

The second condition represents large dissonance between the two types. Even if there is approximately an equal proportion of type-a and type-b individuals, splitting can occur. As with the first condition, this is because the distance between each type and the initial club's agenda is large. The first two conditions have a nice interpretation. Within-club compromise is difficult if there is significant heterogeneity in preferences among members, even if the members have equal power. Now that the extensive margin has been analyzed - whether or not a split will occur, we focus our attention to the timing of the split decision.

Proposition 4. *As $\gamma \to 1$, if there is an incentive to split, the optimal split time is at $t = 1$.*

This result is rather intuitive. If after each period there is full depreciation, then there is no benefit to waiting with regards to splitting so all splits will occur immediately. Thus in order to see strategic membership (agents joining a club with the intent to split), there must be an inter-temporal benefit from contributing.[24] Theorem 1 extends this notion further.

Theorem 1. (Strategic Membership) *Suppose δ is sufficiently large so that individuals are forward looking. If there exists a profitable split at some time $T \geq 0$, there is a threshold $\bar{\gamma} \in [0, 1)$ such that if $\gamma \leq \bar{\gamma}$ and the contribution-scale tension outweighs the agenda-scope tension, then strategic membership occurs.*

[24]Note that if $\gamma = 1$, then the setup is equivalent to the standard repeated club goods game, where in each period, individuals contribute and the capital is used in its entirety such that in the next period, the club starts over.

The intuition follows from an increasing returns argument. If club members are sufficiently patient (a large δ), then it is possible that joining a club that is far from one's preferred agenda, but has a large number of members yields a greater payoff than being a part of a smaller club closer to one's own preferred agenda. This requires that the public benefits outweigh the costs of the club being a poor fit. How large the benefits from are depends on how much contributions depreciate both over time and from the split.

Reconsidering the Bain example, the ten founders of Bain didn't choose to start the company immediately after finishing college. Rather, they joined the preexisting BCG, working within the corporate culture instilled by the company. These individuals didn't perfectly mesh with BCG, having their own ideas (agenda), e.g. the one client per industry, no business cards, no marketing, etc. These were practices not in place at BCG. The individuals began working at BCG developing human capital and thus incubating Bain within BCG. Once enough capital was developed, the individuals were able to split and form Bain.

3.4 Membership Inclusion and Exclusion

Suppose that the club can control who may join and who may not. This leads the concept of strategic admission, where individuals are either permitted to or prevented from joining the club given that this individual will eventually split from the club. Before introducing the result, I define some simplifying notation. Let $V_1(i)$ and $V_2(i)$ represent the indirect utility functions of a member i of a club who do not leave when a split occurs, given that a split occurs at times 1 and 2, respectively. Let $W_1(i)$ and $W_2(i)$ represent the of the indirect utility functions of an individual i who split at times 1 and 2, respectively. Suppose the original club is indexed by $j \in 0$ and the split club by $j \notin 0$.

Theorem 2. (Strategic Admission) *If* $|\int_{j \in 0}(V_2(j) - V_1(j))dj| \geq |\int_{j \notin 0}(W_1(j) - W_2(j))dj|$ *and* $sign\{\int_{j \in 0}(V_2(j) - V_1(j))dj\} = sign\{\int_{j \notin 0}(W_1(j) - W_2(j))dj\}$*, then there exists a value* $\sigma \in [\int_{j \notin 0}(W_1(j) - W_2(j))dj, \int_{j \in 0}(V_2(j) - V_1(j))dj]$ *such that the club offers those who split* σ *to*

(i) Delay their split by 1 *period if* $sign\{\int_{j \in 0}(V_2(j) - V_1(j))dj\} = sign\{\int_{j \notin 0}(W_1(j) - W_2(j))dj\} = 1$.

(ii) Split 1 *period sooner than they otherwise would if* $sign\{\int_{j \in 0}(V_2(j) - V_1(j))dj\} = sign\{\int_{j \notin 0}(W_1(j) - W_2(j))dj\} = -1$.

Theorem 2 is the strategic admission result. Suppose that the optimal time to split is some time $T \geq 1$. Proposition 2 showed that those who practice strategic membership (weakly) increase their marginal contributions prior to splitting. Thus, it may be profitable for the club to keep those members for an extended period of time i.e. an incubation period. If the benefits from allowing the splitting members to incubate are greater than the losses to the splitters when delaying the split by 1 period, then the club can offer a fixed payment, divided amongst the splitters, to delay their decision by 1 period. Thus these individuals are paid to incubate. Similarly, if the splitters would rather wait while the club is worse off with strategic membership, then the club can charge a fee for membership that those who plan on splitting are not willing to pay.

Thus strategic admission can go two ways - either the club can pay to bring individuals in that otherwise wouldn't join or the club can charge a membership fee to keep undesired members out. Under decentralized clubs, where utility is increasing in the number of members, it is likely that incubation will occur, where the club temporarily subsidizes the membership of those who would prefer to form their own club. When clubs are centralized and total contributions may decrease when individuals who are not a good fit join, the club can charge a fee to keep those individuals from joining.

This result hinges upon an implicit assumption - the enforceability of contracts. This requires that, once the fee is paid/subsidy is awarded, neither party rescinds their agreement. This paper offers no insight into possible enforcement mechanisms. However, there is an alternative possibility that requires no commitment mechanism and is more efficient. This is the idea of strategic loss, where, when possible, the club chooses ρ to its benefit.

3.5 Strategic Loss

Strategic loss relies on the properties of incubation. If incubation is preferred, ρ must be set sufficiently low such that there is a large benefit to incubating to build capital. Alternatively, if these members are undesirable, then ρ should be set sufficiently high such that there is no benefit to incubation and those individuals with the desire to split do so immediately. The following result formalizes this.

Theorem 3. **(Strategic Loss)** *The optimal loss* ρ^* *is zero if* $sign\{\int_{j\in 0}(V_2(j) - V_1(j))dj\} = 1$*, and the optimal* ρ^* *is one if* $sign\{\int_{j\in 0}(V_2(j) - V_1(j))dj\} = -1$.

Note that ρ enters only in the utility functions of the group that splits at the time of the split, and does so monotonically. Thus the only possible solutions are boundary solutions. One way to think of an endogenous ρ is to consider intellectual property. Consider, as an example, open source software. The GNU General Purpose License (GNU GPL) requires that all source code be made freely available to anyone who requests it, and further, any modifications made to the code are subject to the GNU GPL conditions (Open Source Initiative, n.d.*b*). On the other hand, the BSD 2-Clause License (BSD) essentially states that any modifications to the code allow the entire code to be considered as proprietary (Open Source Initiative, n.d.*a*). Thus if a group initially producing open source software wants to prevent forking, they can do so selecting the BSD ($\rho^* = 1$) and, in effect, turn the software from open source into proprietary. If the group wants to allow forking, they can select the GNU GPL ($\rho^* = 0$).

3.6 Secondary Splits

Up until this point, we have only looked at one group splitting. Now, suppose that one group - say, type-b individuals, split. What should the type-a individuals do? They could either remain in the original club with agenda $\lambda\alpha + (1-\lambda)\beta$, or form a new group with agenda α. In this case, we slightly alter our interpretation $f(\cdot)$ and the idea of splitting. When there is only a single type remaining, a split can be thought of as a changing of the club's agenda. Thus $f(s)$ represents the cost of changing the agenda.[25] The main result can be summarized as follows.

Proposition 5. *Suppose a split has already occurred, leaving only type-i individuals remaining in the original club. If $f(1) \leq \bar{f}$, then the remaining individuals will pay $f(1)$ to alter the original agenda from $\lambda\alpha + (1-\lambda)\beta$ to A_i.*

[25] This is analytically equivalent to abandoning the original club to form a new club with agenda α.

This has a natural interpretation. If changing the agenda is expensive, there will be a persistent "founder's effect" on the agenda of the club. That is, even though all type-b individuals are no longer in the club, their preference β still factors into the agenda.[26] If the cost of changing the agenda is sufficiently small, then we would expect to see the old club's agenda shift to better match those who remain.

Corporate culture is something that can be seen as expensive to change. It requires a rebranding of the firm and its image. Thus when we see individuals leaving, we are unlikely to see change in the agenda. In politics, we have seen the contrary. After the formation of the Tea Party Movement, many observers reported that the Republican party had changed. For example, former Florida Republican Governor Charlie Crist is quoted saying, "The party just really changed. Or, I should say, the leadership of the party, primarily (Carruthers, 2014)[.]"

3.7 Welfare and Empirical Implications

3.7.1 Welfare Implications

From Proposition 2, it is clear that when a split occurs, those who are splitting increase their contribution levels. Furthermore, from Proposition 3 and Theorem 1, it follows that those who split have a higher utility than if they remained with the original club. This should come as no surprise to the reader - if the utility was lower, then the individuals would opt to not split. The interesting question is what happens to those who remain with the initial club. *A priori*, the utility of those who do not split could move in either direction.

To see this, suppose that type-b individuals split in period 2. The contributions for type-b in period 1 will increase relative to no splitting, while the contributions

[26]Note that I use the "founder's effect" to represent the opposite implication of Ahlerup and Olsson (2012), where it is defined as the lack of variation in new club.

of type-a individuals decrease. Thus the type-a individuals are better off in the first period than if there was not going to be a split. In period 2, there are fewer individuals contributing since all type-b are now gone. Thus each type-a individual must contribute more to receive the same level of utility, which implies period 2 utility is lower. If the gains in period one are larger than the discounted losses in period 2, then splitting is welfare improving. Otherwise, it is unclear (it depends on whether or not type-a losses are smaller than type-b gains).

While it is unclear whether or not the outcome will be Pareto efficient, viz. the first-best outcome, a slightly weaker notion of efficiency will be satisfied. From Theorem 2 and Theorem 3, it is clear that the non-splitting group is able to minimize their losses through the various mechanisms described within. Employing the coalition-proof Nash equilibrium solution concept, which is readily satisfied in this model, the equilibrium outcome will be the most efficient among all self-enforcing agreements (Bernheim et al., 1987).

3.7.2 Empirical Implications

The nature of open source software provides a convenient data source to test the implications of the model. One of the cultural norms of the open source community is attributing credit. Therefore, on every single software update, not only are the changes documented, but identifying information of the individuals who implemented the changes are also documented. Thus using the two most popular online open source repositories, Github and SourceForge, contributor level data can be obtained for each project hosted on the server, which amounts to over 300,000 projects.[27] Each contribution is timestamped, so for any given upload, it is clear both who contributed and exactly how much was contributed by each individual. Furthermore, download

[27] http://www.github.com and http://www.sourceforge.net.

data is also published on the respective repositories. For any given day, the number of downloads is available, sorted by country, etc.

For example, Proposition 2 can be tested by evaluating software that has forked. The first step is to identify an extended period of time both before and after the split occurred, say k months with the split occurring at time T. For each time period $T-k$ to $T-1$, identify contributions from two types of individuals - a group who remained with the project after time T, and a group who joined the fork at time T. Then identify the contribution levels from times $T+1$ to $T+k$. A significant difference between the two within individuals provides support for Proposition 2.

4 Extensions and Future Directions

This framework provides a start to analyzing the endogenous formation and splitting of clubs. There were several assumptions that were made that should be investigated, especially those pertaining to the agenda. Note the agendas enter directly into the utility function, à la Akerlof and Kranton (2000), and was both fixed at the outset and determined exogenously. A more likely scenario is that individuals are endowed with a type, at $t = 0$, which can be assumed fixed. During period 1, members of the community interact and there may in fact be some transmission. Individuals could invest resources to convert their peers. For example, suppose that at the end of period one, a proportion $q(x_i)(1-\lambda)$ of type-j are converted to type-i, where x_i represents the investment in conversion by type-i individuals and $q(\cdot)$ maps from zero to one. Similarly, $q(x_j)\lambda$ type-i individuals are converted to type j.[28]

I interpret this as a reduced form model of cultural transmission. Thus drift could be introduced into the process without the need for extending the model to the infinite

[28]For formal models of cultural transmission, see Bisin and Verdier (2000); Montgomery (2010).

horizon, which leads to several complications due to the many degrees of freedom in the model. This would be most applicable for analyzing long-run processes, such as religious schisms.

5 Concluding Remarks

This paper has shown that seemingly disjoint occurrences, such as religious schisms, political schisms, open source software forks, and the splitting of some firms, can all be explained by considering the endogenous formation of clubs. Not only can these splits be predicted, but in some cases, they can be preferred which provides an explanation for allowing incubation, e.g. in consulting firms, where the turnover in employees is relatively fast.

In particular, the contribution-scale tension and the agenda scope-tension are able to jointly explain the splitting decision on the extensive and intensive margin. Forward looking individuals who are planning on splitting increase their contribution levels prior to splitting in order to take advantage of greater future returns, while those individuals who remain are able to free-ride off of the increased contributions. If the splitting group is a small enough minority, then they may want to practice strategic membership, and delay splitting to take advantage of the increasing returns of a large group. Using the same logic, the majority group may want to delay (or speed up) the split by practicing either strategic admission or strategic loss, depending on the options available. This model also provides empirically testable implications for both changes in contribution levels, as well as changes in overall welfare of the community.

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Appendix

Proof of Proposition 1: Each individual i faces the objective functions described in (2) and (3), which, with respect to $C_2(i)$, yields first-order conditions

$$\frac{\partial}{\partial C_2(i)} G((1-\gamma)C_1^* + C_2^*; A_i - \lambda\alpha - (1-\lambda)\beta) + \frac{\partial}{\partial C_2(i)} L(C_2^*(i); A_i - \lambda\alpha + (1-\lambda)\beta) = F'(C_2^*(i)), \quad \forall i.$$

Note that for each i, the first argument in the functions G and L, are the same, as is the marginal cost. Thus the second argument will determine which individuals contribute more. Without loss of generality, suppose $\lambda \geq \frac{1}{2}$ and recall that G and $\frac{\partial G}{\partial C_t(i)}$ is decreasing in $|A_i - A_k|$. Then $C_2^*(a) \geq C_2^*(b)$ if

$$\begin{aligned} |\alpha - \lambda\alpha - (1-\lambda)\beta| &\leq |\beta - \lambda\alpha - (1-\lambda)\beta)| \\ |(1-\lambda)(\alpha - \beta)| &\leq |\lambda(\alpha - \beta)| \\ 1 - \lambda &\leq \lambda \\ \frac{1}{2} &\leq \lambda. \end{aligned}$$

For the second inclusion, suppose that $C_2^*(a) \geq C_2^*(b)$. Using the above first-order condition, note that the right-hand side will be larger for type-a than type-b. On the left-hand side, the first argument in both G and L are identical among all types, so it must be that the second argument is smaller for type-a, again leading to the above condition on λ. Now, consider $t = 1$. The first-order conditions with respect to $C_1(i)$ are

$$\begin{aligned} \frac{\partial}{\partial C_1(i)} G(C_1^*; A_i - \lambda\alpha - (1-\lambda)\beta) + (1-\gamma)\delta \frac{\partial}{\partial C_2(i)} G((1-\gamma)C_1^* + C_2^*; A_i - \lambda\alpha - (1-\lambda)\beta) \\ + \frac{\partial}{\partial C_1(i)} L(C_1^*(i); A_i - \lambda\alpha - (1-\lambda)\beta) = F'(C_1^*(i)), \quad \forall i. \end{aligned}$$

As before, all differences occur in the second argument. Thus the above breaks down to the same condition as in $t = 2$. Strict convexity in F and weak concavity in G and L guarantee the uniqueness of the symmetric equilibrium. ■

Proof of Proposition 2: Let $C_t^*(i)$ and C_t^* represent equilibrium contribution levels of the original club and let $\hat{C}_t^{**}(i)$ and $\hat{C}_t^{**}$ denote equilibrium contribution levels of the club formed by the split. Suppose that the split occurs at $t = 2$. First, consider the decision at $t = 2$. For each i, the first-order condition with respect to $C_2(i)$ is

$$\frac{\partial}{\partial C_2(i)} G((1 - \rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))(1-\gamma)C_1^{**} + \hat{C}_2^{**}; 0) + \frac{\partial}{\partial C_2(i)} L(\hat{C}_2^{**}(i); 0) = F'(\hat{C}_2^{**}(i)).$$

Notice that the marginal cost is independent of the agendas. Thus the right-hand side remains unchanged. For all values of $C_t(i)$, the marginal benefit of both G and L are larger since $0 < A_i - \lambda\alpha + (1-\lambda)\beta$. Therefore, $(1 - \rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))(1-\gamma)C_1^{**} + \hat{C}_2^{**} > (1-\gamma)C_1^* + C_2^*$. Furthermore, since only $C_2^{**}(i)$ enters on the right-hand side, it must be that $C_2^{**}(i) > C_2^*(i)$, which under a type-symmetric equilibrium implies $\hat{C}_2^{**} > \int_{j \notin 0} C_2^*(j)dj$. Now, consider the period one decision. The first-order condition with respect to $C_1(i)$ is

$$\frac{\partial}{\partial C_1(i)}G(C_1^{**}; A_i - \lambda\alpha - (1-\lambda)\beta) + (1-\gamma)\delta\frac{\partial}{\partial C_2(i)}G((1-\gamma)C_1^{**} + C_2^{**}; 0) + \frac{\partial}{\partial C_1(i)}L(C_1^{**}(i); A_i - \lambda\alpha - (1-\lambda)\beta) = F'(C_1^{**}(i)),$$

for each i. As before, the marginal cost is independent of the agenda. The first term on the left-hand size is identical, as is the third term. However, the second term is strictly larger. Therefore, $C_1^{**}(i) > C_1^*(i)$, which implies $C_1^{**} > C_1^*$. Now suppose that the split occurs at time $t = 1$. The decision at $t = 2$ is identical to the one described above, the second period contributions are strictly larger by each member of the splitting party, and thus of the splitting party as a whole. Lastly, consider the decisions by those who aren't splitting. A non-splitting individual j's first-order condition with respect to $C_2(j)$ is

$$\frac{\partial}{\partial C_2(j)}G((1-\gamma)C_1^{**} + C_2^{**}; A_j - \lambda\alpha - (1-\lambda)\beta) + \frac{\partial}{\partial C_2(j)}L(C_2^{**}(j); A_j - \lambda\alpha + (1-\lambda)\beta) = F'(C_2^{**}(j)).$$

Again, the right-hand side is independent of agenda. The first term on the left-hand side is strictly less than that of an individual splitting. The second term on the left-hand side is also lower for all equal contribution levels. Thus $C_2^{**}(j) < C_2^{**}(i)$ when i splits but j does not. A similar argument holds for $C_1^{**}(j)$ Therefore $\int_{j\in 0} C_t^{**}(j)dj < \int_{j\in 0} C_t^*(j)dj$ for $t = 1, 2$, where $j \in 0$ represents the individuals who do not split. ■

Proof of Proposition 3: The indirect utility of individual i from splitting at time $t = 1$ to form group k is

$$G\left(\int_{j\in k} C_1^{k*}(j)dj; 0\right) + L(C_1^{k*}(i); 0) - F(C_1^{k*}(i)) - f(0) + \delta\left[G\left(\int_{j\in k}((1-\gamma)C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) + L(C_2^{k*}(i); 0) - F(C_2^{k*}(i))\right]. \quad \text{(A.3.1)}$$

The indirect utility for individual i from having no split occur is

$$G\left(C_1^*; A_i - \lambda\alpha - (1-\lambda)\beta\right) + L(C_1^*(i); A_i - \lambda\alpha - (1-\lambda)\beta) - F(C_1^*(i)) + \delta\left[G\left((1-\gamma)C_1^* + C_2^*; A_i - \lambda\alpha - (1-\lambda)\beta\right) + L(C_2^*(i); A_i - \lambda\alpha - (1-\lambda)\beta) - F(C_2^*(i))\right]. \quad \text{(A.3.2)}$$

A split occurs if (A.3.1) > (A.3.2). First, consider condition (*iii*):

$$\begin{aligned} &F(C_1^*(i)) - F(C_1^{k*}(i)) + \delta[F(C_2^*(i)) - F(C_2^{k*}(i))] \\ &\geq G\left(C_1^*; A_i - \lambda\alpha - (1-\lambda)\beta\right) + L(C_1^*(i); A_i - \lambda\alpha - (1-\lambda)\beta) \\ &- G\left(\int_{j\in k} C_1^{k*}(j)dj; 0\right) - L(C_1^{k*}(i); 0) + \delta\Bigg[G\left(\int_{j\in k}((1-\gamma)C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) \\ &+ L(C_2^{k*}(i); 0) - G\left(\int_{j\in k}((1-\gamma)C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) - L(C_2^{k*}(i); 0)\Bigg]. \quad \text{(A.3.3)} \end{aligned}$$

Since both the left-hand side and right-hand side are negative (Proposition 2), we require the left-hand side to be bounded. Since the costs are convex, this is established by bounding the degree of convexity of the left-hand side. Thus, let $\xi_{3,1}$ represent the positive value on the difference in the second derivatives of F such that (A.3.3) holds with equality. To verify existence, note that as F'' increases, $F(x) - F(y)$ increases for $x < y$. Thus an intermediate value theorem argument is sufficient. For (iv) we simply rearrange (A.3.3) such that all of the "G" ("L") functions are on the left-hand side and similarly define $\xi_{4,1}$ as a lower bound on the difference in second derivatives. The argument is essentially identical when the split occurs at $t = 2$, the only difference being in the levels of $\xi_{j,2},\ j = \{1, 2, 3, 4\}$ which satisfy the requirements. ■

Proof of Proposition 4: The proof is straightforward. As $\gamma \to 1$, there is zero benefit to waiting. Thus if a split were to occur, it must at $t = 1$. ■

Proof of Theorem 1: To prove Theorem 1, we compare the payoff from splitting at $t = 2$ to the payoff from $t = 1$. Denote $C_t^{*k}(i)$ as i's contribution to the new club k when the split occurs at $t = 1$ and let $C_t^{**}(i)$ $C_t^{**k}(i)$ represent i's contribution to the original and new club k when the split occurs at $t = 2$. Splitting at $t = 2$ is preferred to $t = 1$ by type-i if

$$G\left(\int_0^1 C_1^{**}(j)dj; A_i - \lambda\alpha - (1-\lambda)\beta\right) + L(C_1^{**}(i); A_i - \lambda\alpha - (1-\lambda)\beta) - F(C_1^{**}(i))$$
$$+ \delta\Bigg[G\left((1 - \rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))(1-\gamma)\int_0^1 C_1^{**}(j)dj + \int_{j\in k} C_2^{k**}(j)dj; 0\right) + L(C_2^{k**}(i); 0)$$
$$- F(C_2^{k**}(i)) - f(0)\Bigg] \geq G\left(\int_{j\in k} C_1^{k*}(j)dj; 0\right) + L(C_1^{k*}(i); 0) - F(C_1^{k*}(i)) - f(0)$$
$$+ \delta\left[G\left(\int_{j\in k}((1-\gamma)C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) + L(C_2^{k*}(i); 0) - F(C_2^{k*}(i))\right].$$

From Proposition 2, it follows that $C_t^{k**}(i) < C_t^{k*}(i)$. Rearranging the above yields

$$G\left(\int_0^1 C_1^{**}(j)dj; A_i - \lambda\alpha - (1-\lambda)\beta\right) - G\left(\int_{j\in k} C_1^{k*}(j)dj; 0\right) + L(C_1^{**}(i); A_i - \lambda\alpha - (1-\lambda)\beta) - L(C_1^{k*}(i); 0)$$
$$+ \delta\Bigg[G\left((1 - \rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))(1-\gamma)\int_0^1 C_1^{**}(j)dj + \int_{j\in k} C_2^{k**}(j)dj; 0\right)$$
$$- G\left(\int_{j\in k}((1-\gamma)C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) + L(C_2^{k**}(i); 0) - L(C_2^{k*}(i); 0)\Bigg]$$
$$- [F(C_1^{**}(i)) + F(C_1^{k*}(i))] - \delta[F(C_2^{k**}(i)) + F(C_2^{k*}(i))] - [f(0) - \delta f(0)] \geq 0$$

First, consider the limit of the above as $\gamma \to 1$:

$$G\left(\int_0^1 C_1^{**}(j)dj; A_i - \lambda\alpha - (1-\lambda)\beta\right) - G\left(\int_{j\in k} C_1^{k*}(j)dj; 0\right) + L(C_1^{**}(i); A_i - \lambda\alpha - (1-\lambda)\beta) - L(C_1^{k*}(i); 0)$$
$$+ \delta\left[G\left(\int_{j\in k} C_2^{k**}(j)dj; 0\right) - G\left(\int_{j\in k} C_2^{k*}(j)dj; 0\right) + L(C_2^{k**}(i); 0) - L(C_2^{k*}(i); 0)\right]$$
$$- [F(C_1^{**}(i)) + F(C_1^{k*}(i))] - \delta[F(C_2^{k**}(i)) + F(C_2^{k*}(i))] - [f(0) - \delta f(0)] < 0$$

Notice that the first line is strictly negative, as is the second and third line. Thus the entire expression is strictly negative. Now consider the case where $\gamma \to 0$:

$$G\left(\int_0^1 C_1^{**}(j)dj; A_i - \lambda\alpha - (1-\lambda)\beta\right) - G\left(\int_{j\in k} C_1^{k*}(j)dj; 0\right) + L(C_1^{**}(i); A_i - \lambda\alpha - (1-\lambda)\beta) - L(C_1^{k*}(i); 0)$$
$$+ \delta\Bigg[G\left((1-\rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))\int_0^1 C_1^{**}(j)dj + \int_{j\in k} C_2^{k**}(j)dj; 0\right)$$
$$- G\left(\int_{j\in k}(C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) + L(C_2^{k**}(i); 0) - L(C_2^{k*}(i); 0)\Bigg]$$
$$- [F(C_1^{**}(i)) + F(C_1^{k*}(i))] - \delta[F(C_2^{k**}(i)) + F(C_2^{k*}(i))] - [f(0) - \delta f(0)]$$

The first line is strictly negative, as is the last two lines. Thus, the middle two lines must be sufficiently positive. Let us isolate lines 2 and 3:

$$+ \delta\Bigg[G\left((1-\rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))\int_0^1 C_1^{**}(j)dj + \int_{j\in k} C_2^{k**}(j)dj; 0\right)$$
$$- G\left(\int_{j\in k}(C_1^{k*}(j) + C_2^{k*}(j))dj; 0\right) + L(C_2^{k**}(i); 0) - L(C_2^{k*}(i); 0)\Bigg]$$

In order for the optimal split time to be $t = 2$, we require

$$(1-\rho(|A_i - \lambda\alpha - (1-\lambda)\beta|))\int_0^1 C_1^{**}(j)dj + \int_{j\in k} C_2^{k**}(j)dj - \int_{j\in k}(C_1^{k*}(j) + C_2^{k*}(j))dj$$

to be sufficiently large such that the public gains outweigh the losses due to poor agenda-fitting. Therefore, for γ sufficiently small, the optimal split time is $t = 2$ if

$$\rho(|A_i - \lambda\alpha - (1-\lambda)\beta|) < 1 + \frac{\int_{j\in k} C_2^{k**}(j)dj - \int_{j\in k}(C_1^{k*}(j) + C_2^{k*}(j))dj - \nu}{\int_0^1 C_1^{**}(j)dj},$$

where $\nu > 0$. The left-hand side provides a measure of the agenda-scope tension while the right-hand side provides a measure of the contribution-scale tension. ■

Proof of Theorem 2: Let $V \equiv \int_{j\in 0}(V_2(j) - V_1(j))dj$ and $W \equiv \int_{j\notin 0}(W_1(j) - W_2(j))dj$, and suppose that $|V| \geq |W|$ and $\text{sign}\{V\} = \text{sign}\{W\}$. This implies that the interval $[W, V]$ exists and is not empty. For (1), suppose that $\text{sign}\{V\} = \text{sign}\{W\} = 1$. $V > 0$ implies that those who do not split prefer the splitters to incubate. $W > 0$ implies that the splitters prefer to leave immediately rather than incubate. Given that $[W, V]$ exists and is nonempty, there exists a value $\bar{\sigma} \in [W, V]$ such that $V - \bar{\sigma} = 0$ and $-W + \bar{\sigma} > 0$. This represents the largest σ that the non-splitters are wiling to offer the splitters to incubate for one period. The smallest value of σ is defined by the conditions $V - \underline{\sigma} > 0$ and $-W + \underline{\sigma} = 0$. This represents the point at which the splitters are indifferent between incubating. Thus a transfer $\sigma \in [V, W]$ is sufficient to delay splitting by one period. The proof is identical for (ii). ■

Proof of Theorem 3: Note that ρ only enters the utility function if a split occurs at $t = 2$. If the original club's utility decreases when a split occurs, then the club can minimize the splitters utility by setting $\rho^* = 1$, thus preventing the split. To the contrary, if splitting is preferred by the original club, then to encourage the behavior, the club can maximize the splitters utility by setting $\rho^* = 0$. ■

Proof of Proposition 5: Suppose the first split has occurred at $t = 1$, and without loss of generality, that type-b individuals initiated the split. Then the indirect utility to a type-a individual if type-a's keep the original agenda is

$$\underbrace{G\left(\int_{i\in a} C_1^*(i)di; (\alpha-\beta)(1-\lambda)\right) + \delta G\left(\int_{i\in a}((1-\gamma)C_1^*(i) + C_2^*(i))di; (\alpha-\beta)(1-\lambda)\right)}_{\equiv G^*}$$
$$+ \underbrace{L(C_1^*(a); (\alpha-\beta)(1-\lambda)) + \delta L(C_2^*(a); (\alpha-\beta)(1-\lambda))}_{\equiv L^*} - \underbrace{F(C_1^*(a)) - \delta F(C_2^*(a))}_{\equiv F^*}.$$

Note that if a split were to occur by the a-types, conditional on a split already occurring by the b-types, then splitting at $t = 1$ strictly dominates splitting at $t = 2$. The payoff from type-a splitting at $t = 1$, given that type-b does as well is

$$\underbrace{G\left(\int_{i\in a} C_1^{**}(i)di; 0\right) + \delta G\left(\int_{i\in a}((1-\gamma)C_1^{**}(i) + C_2^{**}(i))di; 0\right)}_{\equiv G^s}$$
$$+ \underbrace{L(C_1^{**}(a); 0) + \delta L(C_2^{**}(a); 0)}_{\equiv L^s} - \underbrace{F(C_1^{**}(a)) - \delta F(C_2^{**}(a))}_{\equiv F^s} - f(1).$$

Thus

$$\bar{f}_1 \equiv G^s - G^* + L^s - L^* + F^* - F^s.$$

Therefore, if $\bar{f}_1 \geq f(1)$, both types split. A similar approach follows for splits occurring at $t = 2$. ■